A063118 supporting write-up

We consider the ordered list of integers > 1 satisfying gcd(*,30)==1. This list is

7,11,13,17,19,23,29,31,37,41,43,47,49,53,59,61,67,71,73,77,... (1)

The embedded composites seen are 49,77,91,121,...(A038510) and the rest are the primes excepting 2,3,5.

Here we refer to value 7 as being at *position* 1, 41 at *position* 10, etc.

For convenience we label 17 as n=2, with successive n at multiples of 4 values apart in eq'n (1). This gives:

17,31,47,61,77,... for n>=2 (2)

We next consider the sum of ten consecutive values of eq'n (1). **The claim is** A063118 can be generated via this mechanism (for n>3).

For clarity, from *position* 10 the sums begin 228, 264, 300, 336, 372, 412, 450, 488, 528, 564, 600, 636, 672, 712, 750,....

We note that for each **designated n**>=4, the sum is congruent {0} Mod 150.
The sum is of the form 6z+30 for z>=33 (3)

If exactly half of the ten values are 1 Mod 6 and half are 5 Mod 6 (see the proof at the end), there are (3) general solution forms for z seen in eq'n (3) (obtained via WolframAlpha):

In ascending order, z= 75k+45, z=75k+70, z=75(k+1)+20 where k is an integer>=0 (4)

Eq'n (4) generates sums = 6*45+30,6*70+30, 6*95+30,... = 300, 450, 600,... which as stated are seen congruent {0} Mod 150.

This indicates the summing mechanism applied to eq'n (1) will generate A063118 terms for n>=4.

We already have a conjectured formula by Berselli, how does the above relate?

We consider the congruence of terms Mod 6 in eq'n (2) and we see they alternate 5,1,5,1,5,1,... (this was confirmed numerically for 1.33 million terms).

This gives:

Odd n >=3: a(n)= 1 +6*(5*(1+(n-3)/2)) (5)
Even n>=2: a(n)= 5 +6*(2+5*(n-2)/2) (6)

giving $31 + 15(n-3)$ for n odd (7)
$17 + 15(n-2)$ for n even (8)

We can merge (7)&(8) as 24-7(-1)^n + 15*(n-2.5+0.5(-1)^n) (9)

This can be shown to be equivalent to the form stated by Berselli, a(n) = (30*n+(-1)^n-27)/2 for n>1.

The ten value summing mechanism described above was coded in Pari.

The results were checked through 1.33 million terms with 100% confirmation of both the conjectured formulas by Berselli and Librandi:

Librandi's is a(n) = 2*a(n-1) -a(n-2) +2*(-1)^n, n>3.

From (7)&(8) the simpler form is a(n)= a(n-2)+30 n>3, which is Brockhaus'.

As there is no Pari code at A063118, I verified that the summing mechanism terms match the bfile (so ~ 9997 terms) for n>=4.

I have > 1.33 million terms generated via the described mechanism.

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Why *exactly half* of the ten values are 1 Mod 6 and half are 5 Mod 6

We first realize that (1) only contains values congruent {1,5} Mod 6. We also recognize that without any multiples of 3, a list of odd numbers will see multiples of 5 every 5^{th} value for form 6k+1 as well as for form 6k+5. As examples:

6k+1 from k=4: 25,31,37,43,49,55,61,67,73,79,85,...
6k+5 from k=3: 23,29,35,41,47,53,59,65,71,77,83,89,95,...

Since the odd list just described already is without evens and multiples of 3, multiples of 5 are the only way a value can be skipped. Also note that multiples of 5 in form 6k+5 are 10 away from the prior one in form 6k+1, and multiples of 5 in form 6k+1 are 20 away from the prior one in form 6k+5.
This will yield (the starting condition gives no loss of generality)

35= 6(k-1)+5 (multiple of 5)
----------------------------------- summing starts below here
37=6k+1
41=6k+5
43=6(k+1)+1
47=6(k+1)+5
49=6(k+2)+1
53=6(k+2)+5
~~55=6(k+3)+1~~ skip as must be multiple of 5 as = 6k+19 = 6(k-1)+5+20
59=6(k+3)+5

61=6(k+4)+1
~~65=6(k+4)+5~~ -skip as must be multiple of 5 as = 6k+29 = 6(k+3)+1+10
67=6(k+5)+1
71=6(k+5)+5
----------------------------------- summing end above here

The above shows that exactly half of the 10 values being summed are 1 Mod 6, half 5 Mod 6.
What's more, we have the summed values as:

6k+1,6k+5,6(k+1)+1,6(k+1)+5,6(k+2)+1,6(k+2)+5,6(k+3)+5,6(k+4)+1,6(k+5)+1,6(k+5)+5

which is:

60k + 168

And of course this agrees with the 528 sum reported earlier with k=6.

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Basic Pari code

```
genit(maxx=1000)={plst=List();dbg=0;parr=List();bays=List();
forstep(x=7,maxx,2,
if(gcd(x,30)!=1,next);    \\ exclude multiples of 2,3,5
listput(bays,x));  \\end forstep
\\
for(ptr1=10,#bays,
if(ptr1>maxx,break);
\\ we form average of ten consecutive values
sums=0;
for(ptr2=ptr1-9,ptr1,   \\ (as we go, we could just remove oldest value, add newest value ...)
sums=sums+bays[ptr2]);   \\end for ptr2
\\ we only seek sums  congruent {0} Mod 150
if(sums%150==0,listput(plst,ptr1);listput(parr,bays[ptr1]);
if(dbg>0,print( ptr1,"  ",sums);input())
));  \\end for ptr1
\\
print("see list parr for the A063118 sequence n>3");
print("thru term= ",3+#parr );
}
```